MENIIT

NEET | IIT-JEE | FOUNDATION

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JEE MAINS-2019

10-04-2019 Online (Morning)

IMPORTANT INSTRUCTIONS

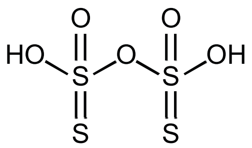
1. The test is of 3 hours duration.
2. This Test Paper consists of **90 questions**. The maximum marks are 360.
3. There are three parts in the question paper A, B, C consisting of **Chemistry, Physics and Mathematics** having 30 questions in each part of equal weightage. Each question is allotted 4 (four) marks for correct response.
4. Out of the four options given for each question, only one option is the correct answer.
5. For each incorrect response 1 mark i.e. ¼ (one-fourth) marks of the total marks allotted to the question will be deducted from the total score. No deduction from the total score, however, will be made if no response is indicated for an item in the Answer Box.
6. Candidates will be awarded marks as stated above in instruction No.3 for correct response of each question. One mark will be deducted for indicating incorrect response of each question. No deduction from the total score will be made if no response is indicated for an item in the answer box.
7. There is only one correct response for each question. Marked up more than one response in any question will be treated as wrong response and marked up for wrong response will be deducted accordingly as per instruction 6 above.

PART-A-CHEMISTRY

1. The oxoacid of sulphur that does not contain bond between sulphur atoms is :

(1*) $H_2S_2O_7$ (2) $H_2S_2O_3$ (3) $H_2S_2O_4$ (4) $H_2S_4O_6$

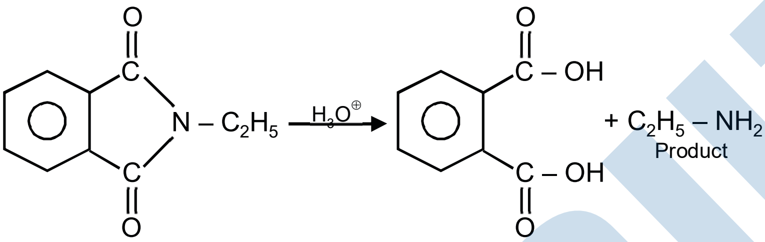
Sol. $H_2S_2O_7$



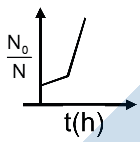
2. Ethylamine ($C_2H_5NH_2$) can be obtained from N-ethylphthalimide on treatment with :

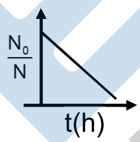
(1*) NH_2NH_2 (2) $NaBH_4$ (3) CaH_2 (4) H_2O

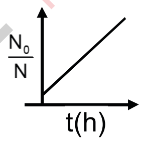
Sol. It is the final step of Gabriel pthalimide synthesis reaction.

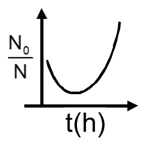


3. A bacterial infection in an internal wound grows as $N'(t) = N_0 \exp(t)$, where the time t is in hours. A dose of antibiotic, taken orally, needs 1 hour to reach the wound. Once it reaches there, the bacterial population goes down as $\frac{dN}{dt} = -5N^2$. What will be the plot of $\frac{N_0}{N}$ vs. t after 1 hour ?

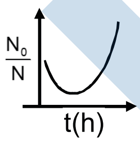
(1) 

(2) 

(3*) 

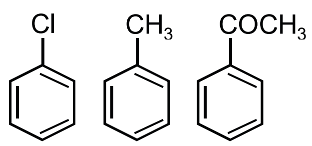
(4) 

Sol. Initially $N > N_0$ and "N" is increasing through first-order kinetics. So N_0/N in initial time decreases.



But after 1 hr the value of N decreases with a faster rate. So N0/N will increases.

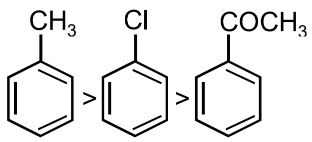
4. The increasing order of the reactivity of the following compounds towards electrophilic aromatic substitution reactions is :-



(I) (II) (III)

(1) II < I < III (2) III < II < I (3) I < III < II (4*) III < I < II

Sol. Rate of aromatic electrophilic substitution is



(i) (ii) (iii)

5. Which of the following is a condensation polymer ?

(1) Buna - S (2*) Nylon 6, 6 (3) Teflon (4) Neoprene

Sol. Nylon-6,6 is a condensation polymer of hexamethylene diamine and adipic acid. Buna-S, Teflon and Neoprene are addition polymer.

6. Three complexes,

$[CoCl(NH_3)_5]^{2+}$ (I), $[Co(NH_3)_5H_2O]^{3+}$ (II) and $[Co(NH_3)_6]^{3+}$ (III)

absorb light in the visible region. The correct order of the wavelength of light absorbed by them is :

(1) (II) > (I) > (III) (2) (III) > (I) > (II) (3) (III) > (II) > (I) (4*) (I) > (II) > (III)

Sol. As we known that

Strong ligand α CFSE α $E_{absorbed}\ \alpha\ \dfrac{1}{\lambda_{absorbed}}$

We have

$[CoCl(NH_3)_5]^{2+}$, $[Co(NH_3)_5H_2O]^{3+}$ and $[CO(NH_3)_6]^{3+}$

(I) (II) (III)

$\therefore$ III > II> I (as per $E_{absorbed}$)

$\therefore\ \lambda_{absorbed}$

I > II > III

7. A gas undergoes physical adsorption on a surface and follows the given Freundlich adsorption isotherm equation

$$\frac{x}{m} = kp^{0.5}$$

Adsorption of the gas increases with :

(1) Decrease in p and increase in T (2*) Increase in p and decrease in T

(3) Decrease in p and decrease in T (4) Increase in p and increase in T

Sol. Increase in pressure leads to the increase in adsorption capacity and the physical adsorption is an exothermic process with the increase in temperature adsorption decrease.

8. A process will be spontaneous at all temperatures if :

(1*) $\Delta H < 0$ and $\Delta S > 0$ (2) $\Delta H > 0$ and $\Delta S > 0$

(3) $\Delta H < 0$ and $\Delta S < 0$ (4) $\Delta H > 0$ and $\Delta S < 0$

Sol. At constant P and T and for the process to be spontaneous we should have ΔG = –ve and we know that

$\Delta G = \Delta H - T\Delta S$

If ΔH = –ve and ΔS = +ve then at all the temperature the process will be spontaneous.

9. The principle of column chromatography is :

(1*) Differential adsorption of the substances on the solid phase.

(2) Gravitational force.

(3) Differential absorption of the substances on the solid phase.

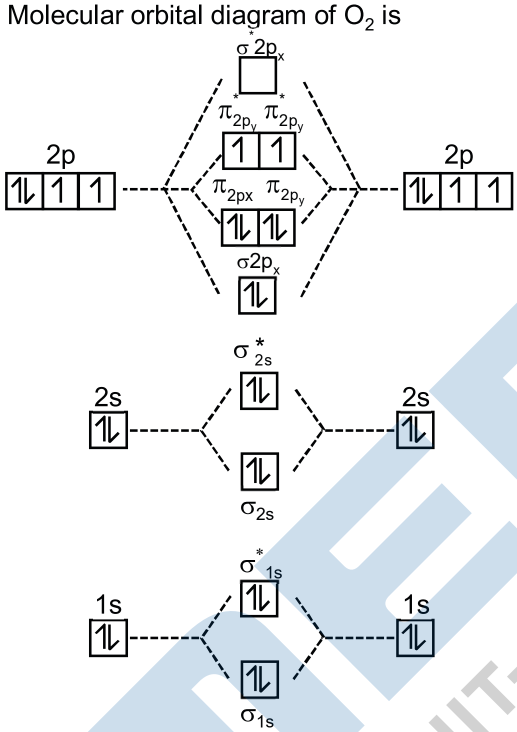
(4) Capillary action.

Sol. The principle of column chromatography is differential adsorption of substance and hence option 1 is correct.

10. During the change of O_2 to O_2^-, the incoming electron goes to the orbital :

(1) $\pi 2p_x$ (2) $\pi 2p_y$ (3*) $\pi^* 2p_x$ (4) $\sigma^* 2p_z$

Sol. Molecular orbital diagram of O_2 is



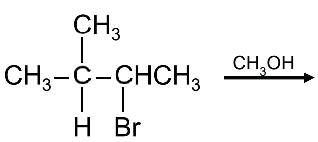
An incoming electron will go in $\sigma^*_{2p_x}$ orbital .

11. The synonym for water gas when used in the production of methanol is :-

(1) fuel gas (2) laughing gas (3) natural gas (4*) syn gas

Sol. The synonoyms for water gas is syn gas.

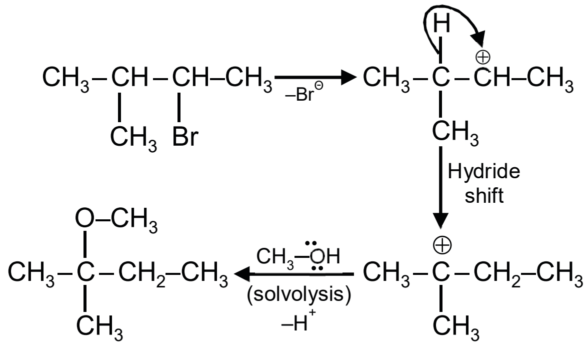
12. The major product of the following reaction is :-



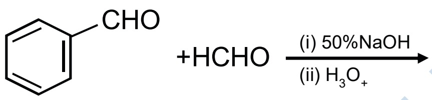
(1*) $CH_3-\underset{OCH_3}{\overset{CH_3}{C}}-CH_2CH_3$ (2) $CH_3-\overset{CH_3}{C}=CHCH_3$

(3) $CH_3-\underset{H}{\overset{CH_3}{C}}-CH=CH_2$ (4) $CH_3-\underset{H}{\overset{CH_3}{C}}-\underset{OCH_3}{CH}CH_3$

Sol. It is S_N1 reaction mechanism is favorable hence reaction complete via S_N1 mechanism



13. Major products of the following reaction are :

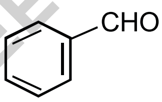


(1) $C_6H_5CH_2OH$ and C_6H_5COOH (2) CH_3OH and HCO_2H

(3*) HCOOH and $C_6H_5CH_2OH$ (4) CH_3OH and C_6H_5COOH

Sol.

This is cross Cannizzaro reaction so more reactive carbonyl compound is oxidized and less reactive

Is reduced so answer is  +HCHO

14. The alloy used in the construction of aircrafts is :-

(1*) Mg – Al (2) Mg – Zn (3) Mg – Sn (4) Mg – Mn

Sol. For aircraft construction aluminum and its alloys are used, because they are lighter.

15. Consider the hydrates ions of Ti^{2+}, V^{2+}, Ti^{3+} and Sc^{3+}. The correct order of their spin-only magnetic moments is :

(1) $V^{2+} < Ti^{2+} < Ti^{3+} < Sc^{3+}$ (2*) $Sc^{3+} < Ti^{3+} < Ti^{2+} < V^{2+}$

(3) $Sc^{3+} < Ti^{3+} < V^{2+} < Ti^{2+}$ (4) $Ti^{3+} < Ti^{2+} < Sc^{3+} < V^{2+]}$

Sol. $Ti^{+2} = 1s^2\ 2s^2\ 2p^6\ 3s^2\ 3p^6\ 3d^2$

Unpaired electrons = 2.

Spin only magnetic moment $(\mu) = \sqrt{2(2+2)} = \sqrt{8}$ B.M

$Ti^{+3} = 1s^2\ 2s^2\ 2p^6\ 3s^2\ 3p^6\ 3d^1$

Unpaired electrons = 1.

$\mu = \sqrt{1(1+2)} = \sqrt{3}$ B.M

$V^{+2} = 1s^2\ 2s^2\ 2p^6\ 3p^6\ 3d^3$

$\mu = \sqrt{3(3+2)} = \sqrt{15}$ B.M

$Sc^{+3} = 1s^2\ 2s^2\ 2p^6\ 3s^2\ 3p^6$

$\mu = 0$

16. The correct order of catenation is :

(1) Ge > Sn > Si > C (2) Si > Sn > C > Ge (3) C > Sn > Si Ge (4*) C > Si > Ge Sn

Ans. In this order of catenation is asked. Catenation is a self-linking property here and for group 14: Self-linking is through covalent bonding

C > Si > Ge ≈ Sn

In C there is 2p – 2p overlapping further 3p – 3p, 4p-4p and so on and the extent of overlapping is more in 2p-2p > 3p-3p > 4p-4p ≈ 5p-5p

17. Consider the statements S1 and S2 :

S1 : Conductivity always increases with decrease in the concentration of electrolyte.

S2 : Molar conductivity always incrteases with decrease in the concentration of electrolyte.

The correct option among the following is :

(1) Both S1 and S2 are wrong (2) S1 is correct and S2 is wrong

(3) Both S1 and S2 are correct (4*) S1 is wrong and S2 is correct

Sol. We know that

$\lambda_m = \frac{K}{C}$ Here λ_m = molar conductivity

K = conductivity

C = concentration

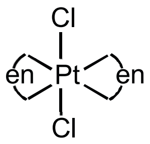
When concentration increase conductivity always increases. The molar conductivity always increase with the decrease in the concentration.

18. The species that can have a trans-isomer is :

(en = ethane-1, 2-diamine, ox = oxalate)

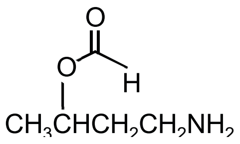
(1) $[Zn(en)Cl_2]$ (2) $[Cr(en)_2(ox)]^+$ (3*) $[Pt(en)_2Cl_2]^{2+}$ (4) $[Pt(en)Cl_2]$

Sol. The trans-isomer of $[Pt(en)_2Cl_2]^{2+}$



19. The major product of the following reaction is :

$$CH_3\underset{}{\overset{OH}{C}}HCH_2CH_2NH_2 \xrightarrow[\text{Triethylamine}]{\text{Ethyl formate (1equiv)}}$$

(1) 

(2) $CH_3-\overset{OH}{C}H-CH=CH_2$

(3*) $CH_3\overset{OH}{C}HCH_2CH_2NHCHO$

(4) $CH_3CH = CH–CH_2NH_2$

Sol. $CH_3–\overset{OH}{C}H–CH_2–CH_2–NH_2 \xrightarrow[\text{Triethylamine}]{\text{ethyl formate (1equiv.)}}$

$CH_3–\overset{OH}{C}H–CH_2–CH_2–NH–C–H$

As NH_2 is a better nucleophile than OH.

20. The regions of the atmosphere, where clouds form and where we live respectively, are :-

(1*) Troposphere and Troposphere

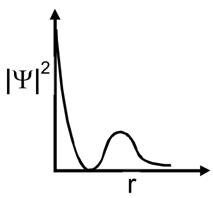
(2) Troposphere and Stratosphere

(3) Stratosphere and Troposphere

(4) Stratosphere and Stratosphere

Sol. Fact based.

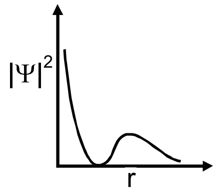
21. The graph between $|\Psi|^2$ and r(radial distance) is shown below. This represents :-



(1*) 2s orbitl (2) 2p orbital (3) 3s orbital (4) 1s orbital

Sol. As we know that for s-orbital graph starts from top and no. of radial node = $n – \lambda –1$

For 2s orbital it will = 2 – 0 – 1 = 1.

$\therefore$ The graph  is of 2s

22. Match the refining methods (Column I) with metals (Column II).

Column I	**Column II**
(Refining methods)	**(Metals)**
(I) Liquation	(a) Zr
(II) Zone Refining	(b) Ni
(III) Mond Process	(c) Sn
(IV) Van Arkel Method	(d) Ga

(1) (I) – (c); (II) – (a); (III) – (b); (IV) – (d)
(2) (I) – (b); (II) – (c); (III) – (d); (IV) – (a)
(3) (I) – (b); (II) – (d); (III) – (a); (IV) – (c)
(4*) (I) – (c); (II) – (d); (III) – (b); (IV) – (a)

Sol. Liquation is used for Sn.

Zone refining is used for Ga.

Mond's process is used for Ni.

Van arkel process is used for Zr.

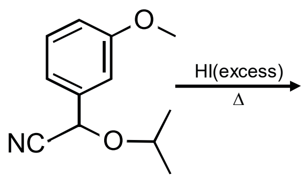
23. The isoelectronic set of ions is :

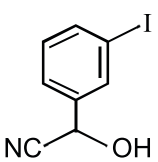
(1) N^{3-}, Li^{+}, Mg^{2+} and O^{2-}
(2) Li^{+}, Na^{+}, O^{2-} and F^{-}
(3*) N^{3-}, O^{2-}, F^{-} and Na^{+}
(4) F^{-}, Li^{+}, Na^{+} and Mg^{2+}

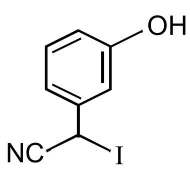
Sol. In this we have to choose isoelectronic set of ions:

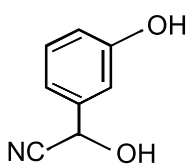
Isoelectronic species are those which have same no. of electron in total, so option 3 is correct.

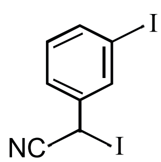
24. The major product of the following reaction is :



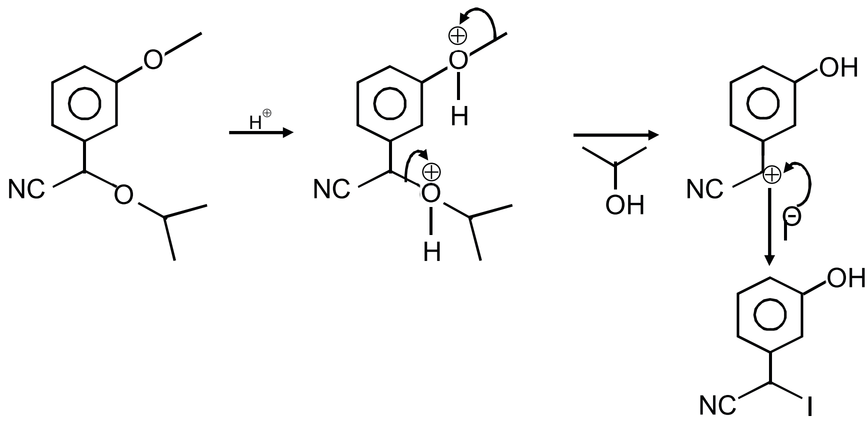
(1) 

(2*) 

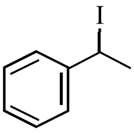
(3) 

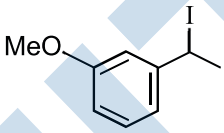
(4) 

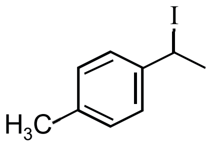
Sol.

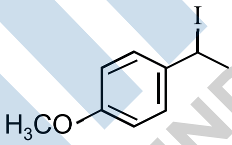


25. Increasing rate of S_N1 reaction in the following compounds is :

(A) 

(B) 

(C) 

(D) 

(1) (A) < (B) < (D) < (C)

(2) (A) < (B) < (C) < (D)

(3) (B) < (A) < (D) < (C)

(4*) (B) < (A) < (C) < (D)

Sol. Rate of S_N1 reaction α stability of $C^{\oplus}$ – I.M

26. At room temperature, a dilute solution of urea is prepared by dissolving 0.60 g of urea in 360 g of water. If the vapour pressure of pure water at this temperature is 35 mmHg, lowering of vapour pressure will be (molar mass of urea = 60 g mol^{-1}):-

(1) 0.027 mmHg (2*) 0.017 mmHg (3) 0.028 mmHg (4) 0.031 mmHg

Sol. lowering of vapour pressure = $P^0 - P = P^0.x_{solute}$

$$\therefore \Delta p = 35 \times \frac{0.6/60}{\frac{0.6}{60} + \frac{360}{18}} = 35 \times \frac{.01}{.01 + 20} = 35 \times \frac{.01}{20.01} = 0.17 \text{ mm Hg}$$

27. Consider the following statements

(a) The pH of a mixture containing 400 mL of 0.1 M H_2SO_4 and 400 mL of 0.1 M NaOH will be approximately 1.3.

(b) Ionic product of water is temperature dependent.

(c) A monobasic acid with $K_a = 10^{-5}$ has a pH = 5. The degree of dissociation of this acid is 50%.

(d) The Le Chatelier's principle is not applicable to common-ion effect.

The correct statements are :

(1) (a) and (b)

(2*) (a), (b) and (c)

(3) (b) and (c)

(4) (a), (b) and (d)

Sol. (a) $H_2SO_4 + 2NaOH \longrightarrow Na_2SO_4 + 2H_2O$

400×.1=40 400×.1=40

$$\therefore \left[H^+\right] = \frac{20\times 2}{800} = \frac{1}{20} \Rightarrow pH = -\log\left(\frac{1}{20}\right)$$

$\therefore$ pH = 1.3 so (a) is correct

$$\text{(b) } \log\left(\frac{Kw_2}{Kw_1}\right) = \frac{\Delta H}{2.303R}\left[\frac{1}{T_1} - \frac{1}{T_2}\right]$$

So ionic product of water is temp. dependent hence (b) is correct.

(c) $K_a = 10^{-5}$, pH = 5 $\Rightarrow$ $[H^+] = 10^{-5}$

$$K_a = \frac{c\alpha^2}{(1-\alpha)} \Rightarrow K_a = \frac{\left[H^+\right].\alpha}{(1-\alpha)}$$

$$\therefore 10^{-5} = \frac{10^{-5}.\alpha}{(1-\alpha)} \Rightarrow 1-\alpha = \alpha \Rightarrow \alpha = \frac{1}{2} = 50\%$$

so (c) is correct.

(d) Le-chatelier's principle is applicable to common-ion effect so option (d) is wrong

28. Consider the following table :

Gas	**a/(k Pa dm^6 mol^{-1})**	**b/(dm^3 mol^{-1})**
A	642.32	0.05196
B	155.21	0.04136
C	431.91	0.05196
D	155.21	0.4382

a and b are vander waals constant. The correct statement about the gases is :

(1) Gas C will occupy lesser volume than gas A; gas B will be more compressible than gas D

(2) Gas C will occupy lesser volume than gas A; gas B will be lesser compressible than gas D

(3) Gas C will occupy more volume than gas A; gas B will be lesser compressible than gas D

(4*) Gas C will occupy more volume than gas A; gas B will be more compressible than gas D

Sol. Gas A and C have same value of 'b' but different value of 'a' so gas having higher value of 'a' have more force of attraction so molecules will be more closer hence occupy less volume. Gas B and D have same value of 'a' but different value of 'b' so gas having lesser value of 'b' will be more compressible.

29. At 300 K and 1 atmospheric pressure, 10 mL of a hydrocarbon required 55 mL of O_2 for complete combustion and 40 mL of CO_2 is formed. The formula of the hydrocarbon is :

(1) C_4H_7Cl

(2) C_4H_{10}

(3) C_4H_8

(4*) C_4H_6

Sol. $C_xH_y + \left(x + \frac{y}{4}\right)O_2 \longrightarrow xCO_2 + \frac{y}{2}H_2O$

10 $\quad 10\left(x + \frac{y}{4}\right) \quad$ 10x

By given data, $10\left(x + \frac{y}{4}\right) = 55$ ……………..(1)

$10x = 40$ ……………..(2)

$\therefore x = 4, y = 6 \Rightarrow C_4H_6$

30. Amylopectin is composed of :

(1) β-D-glucose, $C_1–C_4$ and $C_1–C_6$ linkages

(2) β-D-glucose, $C_1–C_4$ and $C_2–C_6$ linkages

(3) α-D-glucose, $C_1–C_4$ and $C_2–C_6$ linkages

(4*) α-D-Glucose, $C_1–C_4$ and $C_1–C_6$ linkages

Sol. Amylopetcin is a homopolymer of a-D-glucose where $C_1–C_4$ linkage and $C_1 – C_6$ linkage are present.

PART-B-MATHEMATICS

31. If $a > 0$ and $z = \frac{(1+i)^2}{a-i}$, has magnitude $\sqrt{\frac{2}{5}}$, then $\bar{z}$ is equal to

(1*) $-\frac{1}{5}-\frac{3}{5}i$ (2) $-\frac{1}{5}+\frac{3}{5}i$ (3) $-\frac{3}{5}-\frac{1}{5}i$ (4) $\frac{1}{5}-\frac{3}{5}i$

Sol. $z = \frac{(1+i)^2}{a-i} = \frac{2i(a+i)}{a^2+1}$

$|z| = \frac{2}{\sqrt{a^2-i}} = \sqrt{\frac{2}{5}} \Rightarrow a = 3$

$\therefore \bar{z} = \frac{-2i(3-i)}{10}$

$\Rightarrow \frac{-1-3i}{5}$

32. The line x = y touches a circle at the point (1,1). If the circle also passes through the point (1, –3), then its radius is

(1) 3 (2*) $2\sqrt{2}$ (3) $3\sqrt{2}$ (4) 2

Sol. Equation of circle is given as $S + \lambda L = 0$

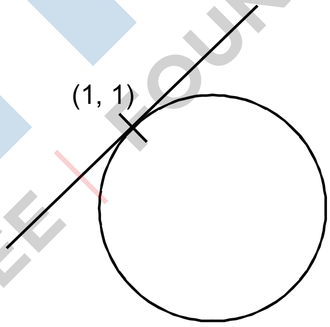
$(x-1)^2 + (y-1)^2 + \lambda(x-y) = 0$ passes

Through (1, –3)

$16 + \lambda \times 4 = 0 \Rightarrow \lambda = -4$

$\therefore (x-1)^2 + (y-1)^2 - 4(x-y) = 0$

$r = 2\sqrt{2}$



33. The value of $\int_0^{2\pi} [\sin 2x(1+\cos 3x)]dx$, where [t] denotes the greatest integer function, is

(1) π (2) -2π (3*) $-\pi$ (4) 2π

Sol. $I = \int_0^{2\pi} [\sin 2x(1+\cos 3x)]dx$

$2I = \int_0^{2\pi} [\sin 2x(1+\cos 3x)] + [\sin 2x - \sin 2x \cos 3x]dx$

$2I = \int_0^{2\pi} -dx$

$2I = \int_0^{\pi} -dx$

$I = \int_0^{\pi} -dx \Rightarrow -\pi$

34. If $a_1, a_2, a_3, \ldots\ldots\ldots, a_n$ are in A.P. and $a_1 + a_4 + a_7 + \ldots\ldots\ldots + a_{16} = 114$, then $a_1 + a_6 + a_{11} + a_{16}$ is equal to

(1) 64 (2) 98 (3*) 76 (4) 38

Sol. $a_1 + a_4 + a_7 + a_{10} + a_{13} + a_{16} = 114$

$\Rightarrow \frac{6}{2}(a_1 + a_{16}) = 144$

$a_1 + a_{16} = 38$

So $a_1 + a_6 + a_{11} + a_{16} = \frac{4}{2}(a_1 + a_{16})$

$= 2 \times 38 \Rightarrow 76$

35. If $f(x) = \begin{cases} \frac{\sin(p+1)x + \sin x}{x} & , x < 0 \\ q & , x = 0 \\ \frac{\sqrt{x + x^2} - \sqrt{x}}{x^{3/2}} & , x > 0 \end{cases}$ is continuous at x = 0, then the ordered pair (p, q) is equal to

(1) $\left(\frac{5}{2}, \frac{1}{2}\right)$ (2) $\left(-\frac{1}{2}, \frac{3}{2}\right)$ (3) $\left(-\frac{3}{2}, -\frac{1}{2}\right)$ (4*) $\left(-\frac{3}{2}, \frac{1}{2}\right)$

Sol. $\text{RHL} = \lim_{x\to\infty} \frac{\sqrt{x + x^2} - \sqrt{x}}{x^{3/2}}$

$= \lim_{x\to\infty} \frac{\sqrt{x + x} - 1}{x} = \frac{1}{2}$

$\text{LHL} = \lim_{x\to\infty} \frac{\sin(\beta + 1)x + \sin x}{x}$

= (p + 1) + 1

= p + 2

For function to be continuous

LHL = RHL = f(0)

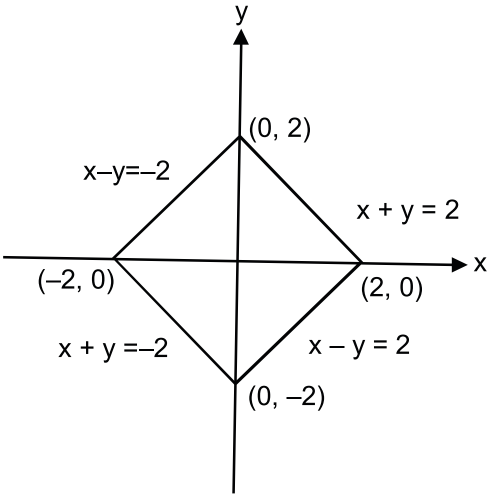
$\Rightarrow (p, q) = \left(\frac{-3}{2}, \frac{1}{2}\right)$

36. The region represented by $|x - y| \le 2$ and $|x + y| \le 2$ is bounded by a

(1) rhombus of area $8\sqrt{2}$ sq.units (2*) square of side length $2\sqrt{2}$ units

(3) square of area 16 sq. units (4) rhombus of side length 2 units

Sol. Shown figure is square with side length $2\sqrt{2}$



37. If for some $x \in R$, the frequency distribution of the marks obtained by 20 students in a test is

Marks	2	3	5	7
Frequency	$(x + 1)^2$	$2x - 5$	$x^2 - 3x$	x

then the mean of the marks is

(1) 3.2 (2*) 2.8 (3) 2.5 (4) 3.0

Sol. Mean $\bar{x} = \frac{\sum x_i f_i}{\sum f_i}$

$\because \Sigma f_i = (x + 1)^2 + (2x - 5) + (x^2 - 3x) + x = 20$

$\Rightarrow x = 3, -4$ (rejected)

$\therefore \bar{x} = \frac{\sum x_i f_i}{\sum f_i} = 2.8$

38. The sum $\frac{3 \times 1^3}{1^2} + \frac{5 \times (1^3 + 2^3)}{1^2 + 2^2} + \frac{7 \times (1^3 + 2^3 + 3^3)}{1^2 + 2^2 + 3^2}$ + ... upto 10^{th} term, is

(1) 680 (2*) 660 (3) 620 (4) 600

Sol. $T_n = \frac{(3 + (n-1) \times 2)(1^3 + 2^3 + ... + n^3)}{(1 + 2^2 + ... + n^2)}$

$= \frac{3}{2} n(n+1)$

$S_n = \Sigma T_n$

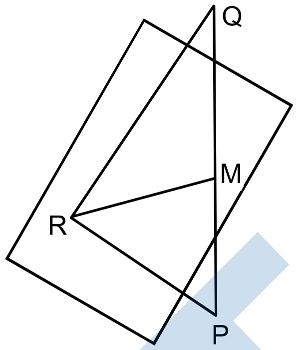
$= \sum \frac{3}{2} .n.(n+1)$

On solving $S_n = \frac{n(n+1)(n+2)}{2} \Rightarrow S_{10} = 660$

39. If Q(0, –1, –3) is the image of the point P in the plane 3x – y + 4z = 2 and R is the point (3, –1, –2), then the area (in sq. units) of ΔPQR is

(1*) $\frac{\sqrt{91}}{2}$ (2) $2\sqrt{13}$ (3) $\frac{\sqrt{91}}{4}$ (4) $\frac{\sqrt{65}}{2}$

Sol. $MQ = \frac{|1-12-2|}{\sqrt{9+1+16}} = \frac{16}{\sqrt{26}} = \sqrt{\frac{13}{2}}$



$PM = \sqrt{26}$

$RQ = \sqrt{9+1} = \sqrt{10}$

$RM = \sqrt{10 - \frac{13}{2}} = \sqrt{\frac{7}{2}}$

$Ar(\Delta PQR) = \frac{1}{2} \times \sqrt{26} \times \sqrt{\frac{7}{2}} = \frac{\sqrt{91}}{2}$

40. If the system of linear equations

x + y + z = 5

x + 2y + 2z = 6

x + 3y + λz = μ, (λ, μ ∈ R), has infinitely many solutions, then the value of λ + μ is

(1) 12 (2) 7 (3*) 10 (4) 9

Sol. x + 3y + λz – u = a(a + y + z – 5) + b(x + 2y + 2z – 6)

Comparing coefficients we get

a + b = 1 and a + 2b = 3

(a, b) = (–1, 2)

So, x + 3y + λz – u = x + 3y +3z – λ

⇒ u = 7, λ = 3

41. All the pairs (x, y) that satisfy the inequality $2^{\sqrt{\sin^2 x - 2\sin x + 5}} \cdot \frac{1}{4^{\sin^2 y}} \le 1$ also satisfy the equation

(1) 2 | sin x | = 3 sin y (2) 2 sin x = sin y (3*) sin x = | sin y | (4) sin x = 2 sin y

Sol. $2^{\sqrt{\sin^2 x - 2\sin x + 5}} . 4^{-\sin^2 y} \le 1$

$\Rightarrow 2^{\sqrt{(\sin x - 1)^2 + 4}} \le 4^{\sin^2 y}$

$\underbrace{\sqrt{\underbrace{(\sin x - 1)^2}_{\ge 0} + 4}}_{\ge 2} \le \underbrace{2\sin^2 y}_{\le 2}$

This is possible only if sin x = 1 and |sin y| = 1

42. If the coefficient of x^2 and x^3 are both zero, in the expansion of the expression $(1 + ax + bx^2)(1 - 3x)^{15}$ in powers of x, then the ordered pair (a, b) is equal to

(1) (– 21, 714) (2*) (28, 315) (3) (– 54, 315) (4) (28, 861)

Sol. Coefficient of $x^2 = {}^{15}C_2 \times 9 - 3a\,({}^{15}C_1) + b = 0$

$\Rightarrow {}^{15}C_2 \times 9 - 45a + b = 0$ (1)

Coefficient of $x^2 = -27 \times {}^{15}C_3 + 9a \times {}^{15}C_2 - 3b \times {}^{15}C_1 = 0$

$\Rightarrow -273 + 21a - b = 0$ (2)

(1) + (2) give

$-24a + 672 = 0 \Rightarrow a = 28, b = 315$

43. If y = y(x) is the solution of the differential equation $\frac{dy}{dx} = (\tan x - y)\sec^2 x,\ x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, such that y(0) = 0, then $y\left(-\frac{\pi}{4}\right)$ is equal to

(1) $2 + \frac{1}{e}$ (2*) $e - 2$ (3) $\frac{1}{e} - 2$ (4) $\frac{1}{2} - e$

Sol. $\frac{dy}{dx} = (\tan x - y)\sec^2 x$

$\frac{dy}{dx} = y\sec^2 x = \tan x \sec^2 x$

Let $\tan x = t \Rightarrow \sec^2 x = \frac{dt}{dx}$

$\therefore \frac{dy}{dt} = (t - y)$

$\frac{dy}{dt} + t = t$ (linear differential equation)

After solving we get

$ye^t = e^t(t - 1) + c$

$\Rightarrow y = (\tan x - 1) + ce^{-\tan x}$

$y(0) = 0 \Rightarrow c = 1$

$y = \tan x - 1 + e^{-\tan x}$

So, $y\left(\frac{-\pi}{4}\right) = e - 2$

44. If $\lim_{x\to 1}\frac{x^4 - 1}{x - 1} = \lim_{x\to k}\frac{x^3 - k^3}{x^2 - k^2}$, then k is

(1) $\frac{4}{3}$ (2*) $\frac{8}{3}$ (3) $\frac{3}{8}$ (4) $\frac{3}{2}$

Sol. $\lim_{x\to 1}\frac{x^4 - 1}{x - 1} = \lim_{x\to 1}(x + 1)(x^2 + 1)$..........(1)

$\lim\limits_{x \to k} \frac{x^3 - k^3}{x^2 - k^2} = \frac{k^2 + k^2 + k^2}{2k}$..........(2)

(1) = (2)

$\Rightarrow k = \frac{8}{3}$

45. Which one of the following Boolean expressions is a tautology?

(1) $(p \vee q) \wedge (p \vee \sim q)$ (2*) $(p \vee q) \vee (p \vee \sim q)$

(3) $(p \vee q) \wedge (\sim p \vee \sim q)$ (4) $(p \vee q) \vee (p \wedge \sim q)$

Sol. From options

$(p \vee q) \wedge (\sim p \vee \sim q) = (p \vee q) \wedge \sim (p \wedge q) \to$ Not a tautology

$(p \vee q) \vee (p \vee \sim q) = p \vee (q \vee \sim q) \to$ tautology

$(p \wedge q) \vee (p \wedge \sim q) \equiv p \wedge (q \vee \sim q) \to$ Not a tautology

$(p \vee q) \wedge (p \vee \sim q) \equiv p \vee (q \wedge \sim q) \to$ Not a tautology

46. Let $f(x) = x^2$, $x \in R$. For any $A \subseteq R$, define $g(A) = \{x \in R : f(x) \in A\}$. If $S = [0, 4]$, then which one of the following statements is not true?

(1) $f(g(S)) = S$ (2*) $g(f(S)) = g(S)$ (3) $f(g(S)) \neq f(S)$ (4) $g(f(S)) \neq S$

Sol. $g(s) = [-2, 2]$

$f(g(s)) = [0, 4] = 5$

$f(S) = [0, 16] \Rightarrow f(g(s)) \neq f(s)$

$g(f(s)) = [-4, 4] \neq g(s)$

therefore, $g(f(s)) \neq S$

47. If α and β are the roots of the quadratic equation, $x^2 + x \sin\theta - 2\sin\theta = 0$, $\theta \in \left(0, \frac{\pi}{2}\right)$, then

$\frac{\alpha^{12} + \beta^{12}}{(\alpha^{-12} + \beta^{-12}) \cdot (\alpha - \beta)^{24}}$ is equal to

(1*) $\frac{2^{12}}{(\sin\theta + 8)^{12}}$ (2) $\frac{2^{12}}{(\sin\theta - 4)^{12}}$ (3) $\frac{2^{12}}{(\sin\theta - 8)^{6}}$ (4) $\frac{2^{6}}{(\sin\theta + 8)^{12}}$

Sol. $x^2 + x\sin\theta - 2\sin\theta = 0$

$\alpha + \beta = -\sin\theta$

$\alpha\beta = -2\sin\theta$

Now, $\frac{\alpha^{12} + \beta^{12}}{\left(\frac{1}{\alpha^{12}} + \frac{1}{\beta^{12}}\right)(\alpha - \beta)^{24}} = \frac{(\alpha\beta)^{12}}{(\alpha - \beta)^{24}}$

$= \frac{(\alpha\beta)^{12}}{\left((\alpha + \beta)^2 - 4\alpha\beta\right)^{12}}$

$$= \left[\frac{\alpha\beta}{(\alpha+\beta)^2 - 4\alpha\beta}\right]^{12}$$

$$= \left(\frac{-2\sin\theta}{\sin^2\theta + 8\sin\theta}\right)^{12}$$

$$= \frac{2^{12}}{(\sin\theta + 8)^{12}}$$

48. Let A(3, 0, –1), B(2, 10, 6) and C(1, 2, 1) be the vertices of a triangle and M be the midpoint of AC. If G divides BM in the ratio, 2 : 1, then cos (∠GOA) (O being the origin) is equal to

(1) $\frac{1}{2\sqrt{15}}$ (2) $\frac{1}{\sqrt{30}}$ (3*) $\frac{1}{\sqrt{15}}$ (4) $\frac{1}{6\sqrt{10}}$

Sol. G will be centroid of ΔABC

$G \equiv (2, 4, 2)$

$\overrightarrow{OG} = 2\hat{i} + 4\hat{j} + 2\hat{k}$

$\overrightarrow{OA} = 3i - \hat{k}$

$$\cos(\angle GOA) = \frac{\overrightarrow{OG}.\overrightarrow{OA}}{|\overrightarrow{OG}||\overrightarrow{OA}|} = \frac{1}{\sqrt{15}}$$

49. If a directrix of a hyperbola centred at the origin and passing through the point $(4, -2\sqrt{3})$ is $5x = 4\sqrt{5}$ and its eccentricity is e, then

(1) $4e^4 - 24e^2 + 27 = 0$ (2*) $4e^4 - 24e^2 + 35 = 0$

(3) $4e^4 - 12e^2 - 27 = 0$ (4) $4e^4 + 8e^2 - 35 = 0$

Sol. Let hyperbola be $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ and passess through $(4, -2\sqrt{3})$ therefore

$\frac{16}{a^2} - \frac{12}{b^2} = 1$ …………..(1)

$\because b^2 = a^2(e^2 - 1)$

$x = \frac{4\sqrt{5}}{5} = \frac{a}{e} \Rightarrow a^2 = \frac{16}{5}e^2$ ………..(2)

On solving (i) and (ii)

$\Rightarrow 4e^4 - 24e^2 + 35 = 0$

50. If $\Delta_1 = \begin{vmatrix} x & \sin\theta & \cos\theta \\ -\sin\theta & -x & 1 \\ \cos\theta & 1 & x \end{vmatrix}$ and $\Delta_2 = \begin{vmatrix} x & \sin 2\theta & \cos 2\theta \\ -\sin 2\theta & -x & 1 \\ \cos 2\theta & 1 & x \end{vmatrix}, x \neq 0$; then for all $\theta \in \left(0, \frac{\pi}{2}\right)$

(1) $\Delta_1 + \Delta_2 = -2(x^3 + x - 1)$ (2*) $\Delta_1 + \Delta_2 = -2x^3$

(3) $\Delta_1 - \Delta_2 = -2x^3$ (4) $\Delta_1 - \Delta_2 = x(\cos 2\theta - \cos 4\theta)$

Sol. $\Delta_1 = \begin{vmatrix} x & \sin\theta & \cos\theta \\ -\sin\theta & -x & 1 \\ \cos\theta & 1 & x \end{vmatrix}$

$= x(-x^2-1) - \sin\theta(-x\sin\theta - \cos\theta) + \cos\theta(-\sin\theta + x\cos\theta) \Rightarrow -x^3$

$\Delta_2 = \begin{vmatrix} x & \sin 2\theta & \cos 2\theta \\ -\sin 2\theta & -x & 1 \\ \cos 2\theta & 1 & x \end{vmatrix}$

$\Rightarrow -x^3$

$\Delta_1 + \Delta_2 = -2x^3$

51. Let f (x) = $e^x - x$ and g(x) = $x^2 - x$, $\forall x \in R$. Then the set of all $x \in R$, where the function h(x) = (fog) (x) is increasing, is

(1) $\left[-1, \frac{-1}{2}\right] \cup \left[\frac{1}{2}, \infty\right)$ (2) $[0, \infty)$

(3) $\left[\frac{-1}{2}, 0\right] \cup [1, \infty)$ (4*) $\left[0, \frac{1}{2}\right] \cup [1, \infty)$

Sol. h(x) = f(g(x))

$\therefore$ h'(x) = f'(g(x))g'(x) and f'(x) = $e^x - 1$

h'(x) = $(e^{g(x)} - 1)$ g'(x)

$h'(x) = \left(e^{x^2-x} - 1\right)(2x-1) \geq 0$

Case : 1

$e^{x^2-x} \leq 1$ and $2x - 1 \geq 0$

$\Rightarrow x \in [1, \infty)$ ....(1)

Case : 2

$e^{x^2-x} \geq 1$ and $2x - 1 \geq 0$

$\Rightarrow x \in [1, \infty)$ ....(2)

From (i) and (ii)

$x \in \left[0, \frac{1}{2}\right] \cup [1, \infty)$

52. ABC is a triangular park with AB = AC = 100 metres. A vertical tower is situated at the mid-point of BC. If the angles of elevation of the top of the tower at A and B are $\cot^{-1}(3\sqrt{2})$ and $\text{cosec}^{-1}(2\sqrt{2})$ respectively, then the height of the tower (in metres) is

(1) 25 (2) $10\sqrt{5}$ (3*) 20 (4) $\frac{100}{3\sqrt{3}}$

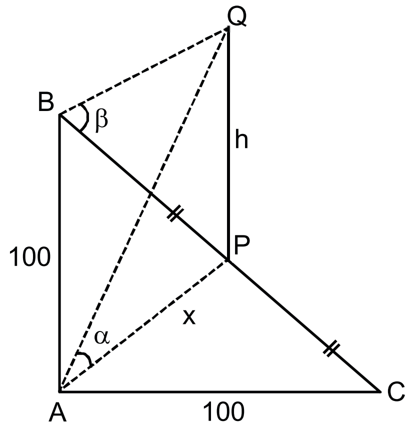
Sol. $\text{cosec}\beta = 2\sqrt{2}$

$\cot\alpha = 3\sqrt{2}$

$\frac{x}{h} = 3\sqrt{2}$.........(i)

So $\dfrac{\alpha}{\sqrt{10^4 - x^2}} = \dfrac{1}{\sqrt{7}}$.........(ii)

From (i) and (ii)



h = 20

53. If $\int \dfrac{dx}{(x^2 - 2x + 10)^2} = A\left(\tan^{-1}\left(\dfrac{x-1}{3}\right) + \dfrac{f(x)}{x^2 - 2x + 10}\right) + C$

where C is a constant of integration, then

(1) $A = \dfrac{1}{81}$ and $f(x) = 3(x - 1)$

(2) $A = \dfrac{1}{54}$ and $f(x) = 9(x - 1)^2$

(3) $A = \dfrac{1}{27}$ and $f(x) = 9(x - 1)$

(4*) $A = \dfrac{1}{54}$ and $f(x) = 3(x - 1)$

Sol. $\int \dfrac{dx}{(x^2 - 2x + 10)^2} = \int \dfrac{dx}{((x-1)^{21} + 9)^2}$

Let $x - 1 = 3\tan\theta$

$dx = 3\sec^2\theta d\theta$

$\therefore \dfrac{1}{27}\int \cos^2\theta d\theta = \dfrac{1}{54}\int(1 + \cos 2\theta)d\theta = \dfrac{1}{54}\left(\theta + \dfrac{\sin 2\theta}{2}\right)$

$= \dfrac{1}{54}\left(\tan^{-1}\left(\dfrac{x-1}{3}\right) + \dfrac{3(x-1)}{x^2 - 2x + 10}\right) + C$

54. Assume that each born child is equally likely to be a boy or a girl. If two families have two children each, then the conditional probability that all children are girls given that at least two are girls is

(1) $\dfrac{1}{12}$ (2) $\dfrac{1}{10}$ (3*) $\dfrac{1}{11}$ (4) $\dfrac{1}{17}$

Sol. $P(\text{Boy}) = P(\text{girl}) = \dfrac{1}{2}$

Required probability $= \dfrac{\text{all four girls}}{\text{Atleast two girls}}$

$$= \frac{\left(\frac{1}{2}\right)^4}{\left(\frac{1}{2}\right)^4 + {}^4C_3\left(\frac{1}{2}\right)^4 + {}^4C_3\left(\frac{1}{2}\right)^4}$$

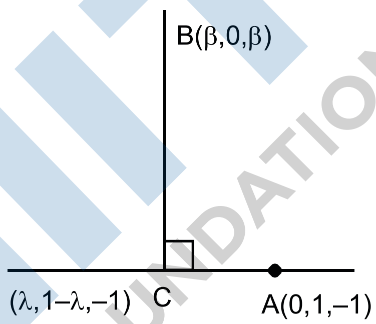
$$= \frac{1}{11}$$

55. If the length of the perpendicular from the point (β, 0, β) (β ≠ 0) to the line, $\frac{x}{1} = \frac{y-1}{0} = \frac{z+1}{-1}$ is $\sqrt{\frac{3}{2}}$, then β is equal to

(1) 2 (2) –2 (3*) –1 (4) 1

Sol. $\frac{x}{1} = \frac{y-1}{0} = \frac{z+1}{-1} = \lambda$

A point on this line is A(0, 1, –1)



$\overrightarrow{AB}.\overrightarrow{BC} = 0$

We get $\lambda = \frac{-1}{2}$

$\therefore C = \left(-\frac{1}{2}, 1, \frac{-1}{2}\right)$

$\left|\overrightarrow{BC}\right| = \sqrt{\frac{2}{3}}$

$$\sqrt{\left(\beta + \frac{1}{2}\right)^2 + \left(1^2 + \left(\beta + \frac{1}{2}\right)\right)^2} = \sqrt{\frac{2}{3}}$$

∴ β = 0, –1

β = –1 (β ≠ 0)

56. Let f : R → R be differentiable at c ∈ R and f (c) = 0. If g(x) = | f(x) |, then at x = c, g is

(1) not differentiable (2) differentiable if $f'(c) \neq 0$

(3*) differentiable if $f'(c) = 0$ (4) not differentiable if $f'(c) = 0$

Sol. $g'(c) = \lim_{x \to 0} \frac{|f(c+h)| - |f(c)|}{h}$ ∵ f(c) = 0

$= \lim_{x \to 0} \frac{|f(c+h)|}{h}$

$= \lim_{x \to 0} = \left|\frac{f(c+h) - f(c)}{h}\right| \cdot \frac{|h|}{h}$

$= \lim_{x \to 0} = |f'(c)| \frac{|h|}{h} = 0 \text{ if } f'(c) = 0$

i.e. g(x) is differentiable at x = c if f'(c)=0

57. If the circles $x^2 + y^2 + 5kx + 2y + K = 0$ and $2(x^2 + y^2) + 2Kx + 3y - 1 = 0$, $(K \in R)$, intersect at the points P and Q, then the line $4x + 5y - K = 0$ passes through P and Q, for

(1) exactly two values of K
(2) exactly one value of K
(3) infinitely many values of K
(4*) no value of K

Sol. Equation of common chord $4kx + \frac{1}{2}y + k + \frac{1}{2} = 0$............(i)

and given line $4x + 5y - k = 0$ .....(ii)

on comparing (i) and (ii) we get $k = \frac{1}{10} = \frac{k + \frac{1}{2}}{-k}$

$\Rightarrow$ No real value of k exist.

58. $\lim_{n \to \infty} \left(\frac{(n+1)^{1/3}}{n^{4/3}} + \frac{(n+2)^{1/3}}{n^{4/3}} + ... + \frac{(2n)^{1/3}}{n^{4/3}} \right)$ is equal to

(1*) $\frac{3}{4}(2)^{4/3} - \frac{3}{4}$
(2) $\frac{3}{4}(2)^{4/3} - \frac{4}{3}$
(3) $\frac{4}{3}(2)^{4/3}$
(4) $\frac{4}{3}(2)^{3/4}$

Sol. $\lim_{x \to \infty} \sum_{r=1}^{n} \frac{1}{n} \left(\frac{n+r}{n} \right)^{1/3}$

$= \int_0^1 (1+x)^{1/3} dx = \frac{3}{4}\left(2^{4/3} - 1\right)$

59. The number of 6 digit numbers that can be formed using the digits 0, 1, 2, 5, 7 and 9 which are divisible by 11 and no digit is repeated, is

(1) 36
(2) 48
(3) 72
(4*) 60

Sol. Let the six digit number be abcdef for this number to be divisible by 11

$(a + c + e) - (b + d + f)$ must be multiple of 11

$\therefore$ possibility is $a + c + c = b + d + f = 12$

Case I {a,c,e} = {7,5,0} and {b,d,f} = {9,2,1}

So, number of numbers = $2 \times 2! \times 3! = 24$

Case II {a,c,e} = {3,2,1} {b,d,f} = {7,5,0}

So number of numbers = $3! \times 3! = 36$

Total $\Rightarrow 24 + 36 = 60$

60. If the line $x - 2y = 12$ is tangent to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ at the point $\left(3, \frac{-9}{2}\right)$, then the length of the latus rectum of the ellipse is

(1) $8\sqrt{3}$
(2) $12\sqrt{2}$
(3) 5
(4*) 9

Sol. Tangent at $\left(3, \frac{-9}{2}\right)$

$$\frac{3x}{a^2} - \frac{9y}{2b^2} = 1$$

Comparing with x – 2y = 12

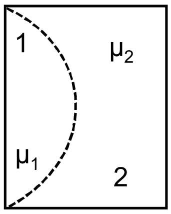
$$\frac{3}{a^2} - \frac{9}{4b^2} = \frac{1}{12}$$

$\Rightarrow a = 6$ and $b = 3\sqrt{3}$

Length of latus rectum $= \frac{2b^2}{a} = 9$

PART-C-PHYSICS

61. One plano-convex and one plano-concave lens of same radius of curvature 'R' but of different materials are joined side by side as shown in the figure. If the refractive index of the material of 1 is μ_1 and that of 2 is μ_2, then the focal length of the combination is :



(1) $\frac{R}{2(\mu_1-\mu_2)}$ (2*) $\frac{R}{\mu_1-\mu_2}$ (3) $\frac{2R}{\mu_1-\mu_2}$ (4) $\frac{R}{2-(\mu_1-\mu_2)}$

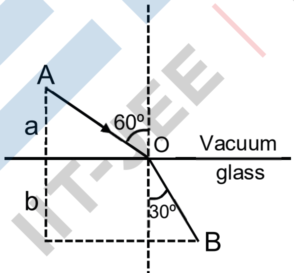
Sol. For 1st lens $\frac{1}{f_1}=\left(\frac{\mu-1}{1}\right)\left(\frac{1}{\infty}-\frac{1}{-R}\right)=\frac{\mu_1-1}{R}$

For 2st lens $\frac{1}{f_2}=\left(\frac{\mu_2-1}{1}\right)\left(\frac{1}{-R}-0\right)=\frac{\mu_2-1}{R}$

$\frac{1}{f_{eq}}=\frac{1}{f_1}+\frac{1}{f_2}$

$\frac{1}{f_{eq}}=\frac{\mu_1-1}{R}+\frac{-(\mu_2-1)}{-R}\Rightarrow f_{eq}=\frac{R}{\mu_1-\mu_2}$

62. A ray of light AO in vacuum is incident on a glass slab at angle 60° and refracted at angle 30° along OB as shown in the figure. The optical path length of light ray from A to B is :



(1) $2a+\frac{2b}{3}$ (2) $2a+\frac{2b}{\sqrt{3}}$ (3*) 2a + 2b (4) $\frac{2\sqrt{3}}{a}+2b$

Sol. From Snell's law

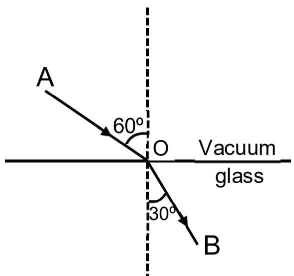
1 sin 60° = μ sin 30°

$\Rightarrow \mu=\sqrt{3}$

Optical path = AO + μ(OB)

$=\frac{a}{\cos 60^\circ}+\sqrt{3}\frac{b}{\cos 30^\circ}$

= 2a + 2b



63. In a photoelectric effect experiment the threshold wavelength of the light is 380 nm. If the wavelength of incident light is 260 nm, the maximum kinetic energy of emitted electrons will be:

Given E (in eV) = $\frac{1237}{\lambda(\text{in nm})}$

(1) 15.1 eV (2) 3.0 eV (3) 4.5 eV (4*) 1.5 eV

Sol. $K_{max} = \frac{hc}{\lambda} - \frac{hc}{\lambda_0}$

$\Rightarrow K_{max} = hc\left(\frac{\lambda_0 - \lambda}{\lambda\lambda_0}\right)$

$\Rightarrow K_{max} = (1237)\left(\frac{380-260}{380\times 260}\right)$

= 1.5 eV

64. A transformer consisting of 300 turns in the primary and 150 turns in the secondary gives output power of 2.2 kW. If the current in the secondary coil is 10A, then the input voltage and current in the primary coil are :

(1) 220 V and 20 A (2) 440 V and 20 A (3) 220 V and 10 A (4*) 440 V and 5 A

Sol. Given N_P = 300, N_s = 150, P_0 = 2200W

I_s = 10 A

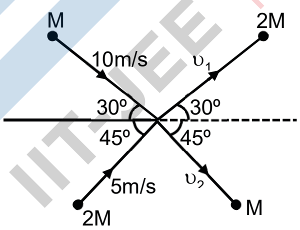
$P_0 = V_0I_0 \Rightarrow 2200 = V_0 \times 10 \Rightarrow$ V0 = 220 V

$\because \frac{V_i}{V_0} = \frac{N_P}{N_s} \Rightarrow V_i = 2\times 200 = 440$ V

Also, P0 = V_iI_i

$\Rightarrow I_i = \frac{2200}{440} = 5A$

65. Two particles, of masses M and 2M, moving, as shown, with speeds of 10 m/s and 5 m/s, collide elastically at the origin. After the collision, they move along the indicated directions with speeds υ_1 and υ_2, respectively. The values of υ_1 and υ_2 are nearly :



(1*) 6.5 m/s and 6.3 m/s (2) 3.2 m/s and 6.3 m/s
(3) 3.2 m/s and 12.6 m/s (4) 6.5 m/s and 3.2 m/s

Sol. 2MV, cos 30° + Mv_2 cos 45° = 10 M cos 30° + 10 cos 45°

$\Rightarrow v_1\sqrt{3} + \frac{v_2}{\sqrt{2}} = 5\sqrt{3} + 5\sqrt{2}\ldots(i)$

2MV, sin 30° – MV_2 sin 45° = –10 M sin 30° + 10 M sin 45°

$V_1 = \frac{5\left(\sqrt{3}-1\right)+10\sqrt{2}}{\sqrt{3}+1} = \frac{17.5}{2.7} = 6.5\ m/s$

V_2 = 6.3 m/s

66. A particle of mass m is moving along a trajectory given by

$x = x_0 + a\cos\omega_1 t$

$y = y_0 + b\sin\omega_2 t$

The torque, acting on the particle about the origin, at t = 0 is:

(1) Zero

(2) $-m(x_0 b\omega_2^2 - y_0 a\omega_1^2)\hat{k}$

(3*) $+my_0 a\omega_1^2\hat{k}$

(4) $m(-x_0 b + y_0 a)\omega_1^2\hat{k}$

Sol. $\vec{F} = m\vec{a} = m[-a\omega_1^2 \cos\omega_1 t\hat{i} - b\omega_2^2 \sin\omega_2 t\hat{j}$

$\vec{f}_{t=0} = -ma\omega_1^2\hat{i}$

$\vec{r}_{t=0} = (x_0 + a)\hat{i} + y\hat{j}$

$\vec{\tau} = \vec{r} \times \vec{F} = my_0 a\omega_1^2\hat{k}$

67. Two coaxial discs, having moments of inertia I_1 and $\frac{I_1}{2}$, are rotating with respective angular velocities ω_1 and $\frac{\omega_1}{2}$, about their common axis. They are brought in contact with each other and thereafter they rotate with a common angular velocity. If E_f and E_i are the final and initial total energies, then $(E_f - E_i)$ is:

(1) $-\frac{I_1\omega_1^2}{12}$

(2) $\frac{I_1\omega_1^2}{6}$

(3) $\frac{3}{8}I_1\omega_1^2$

(4*) $-\frac{I_1\omega_1^2}{24}$

Sol. $E_i = \frac{1}{2}I_i \times \omega_1^2 + \frac{1}{2}\frac{I}{2} \times \frac{\omega_1^2}{4}$

$= \frac{I_1\omega_1^2}{2}\left(\frac{9}{8}\right) = \frac{9}{16}I_1\omega_1^2$

$I_1\omega_1 + \frac{I_1\omega_1}{4} = \frac{3I_1}{2}\omega \quad ; \quad \frac{5}{4}I_1\omega_1 = \frac{3I_1}{2}\omega$

$\omega = \frac{5}{6}\omega_1 \quad ; \quad E_f = \frac{1}{2} \times \frac{3I}{2} \times \frac{25}{36}\omega_1^2$

$= \frac{25}{48}I_1\omega_1^2$

$\Rightarrow E_f - E_i = I_1\omega_1^2\left(\frac{25}{48} - \frac{9}{16}\right)\frac{-2}{48}I_1\omega_1^2$

$= \frac{-I_1\omega_1^2}{24}$

68. The value of acceleration due to gravity at Earth's surface is 9.8 ms^{-2}. The altitude above its surface at which the acceleration due to gravity decreases to 4.9 ms^{-2}, is close to :

(Radius of earth = 6.4×10^6 m)

(1*) 2.6×10^6 m

(2) 1.6×10^6 m

(3) 6.4×10^6 m

(4) 9.0×10^6 m

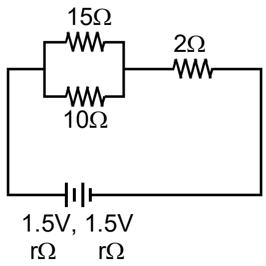
Sol. $\frac{GM}{(R+h)^2} = \frac{GM}{2R^2}$

$R + h = \sqrt{2}R$

$h = \left(\sqrt{2} - 1\right)R$

 2.6 × 10^6 m

69. In the given circuit, an ideal voltmeter connected across the 10Ω resistance reads 2V. The internal resistance r, of each cell is :



(1*) 0.5Ω (2) 1Ω (3) 1.5Ω (4) 0Ω

Sol. $R_{eq} = \frac{15 \times 10}{25} + 2 + 2r$

= 8 + 2r

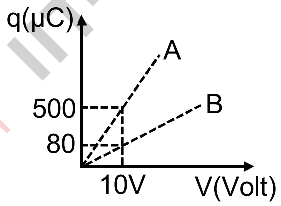
$i = \frac{3}{8 + 2r}$

$2 = iR_{eq} = \frac{3}{8 + 2r} \times 6$

16 + 4r = 18

⇒ r = 0.5 Ω

70. Figure shows charge (q) versus voltage (V) graph for series and parallel combination of two given capacitors. The capacitances are :



(1) 50 μF and 30 μF (2*) 40 μF and 10 μF (3) 20 μF and 30 μF (4) 60 μF and 40 μF

Sol. As q = CV

Hence slope of graph will give capacitance. Slope will be more in parallel combination.

Hence capacitance in parallel should be 50 μF and in series combination must be 8 μF.

Only in option 40 μf and 10 μF.

71. The electric field of a plane electromagnetic wave is given by

$\vec{E} = E_0 \hat{i} \cos(kz)\cos(\omega t)$

The corresponding magnetic field $\vec{B}$ is then given by :

(1) $\vec{B}=\frac{E_0}{C}\hat{j}\cos(kz)\sin(\omega t)$ (2) $\vec{B}=\frac{E_0}{C}\hat{k}\sin(kz)\cos(\omega t)$

(3) $\vec{B}=\frac{E_0}{C}\hat{j}\sin(kz)\cos(\omega t)$ (4*) $\vec{B}=\frac{E_0}{C}\hat{j}\sin(kz)\sin(\omega t)$

Sol. $\because\ \vec{V}\times\vec{E}\parallel\vec{v}$

Given that wave is propagating along positive z-axis and $\vec{E}$ along positive x-axis. Hence $\vec{B}$ along y-axis.

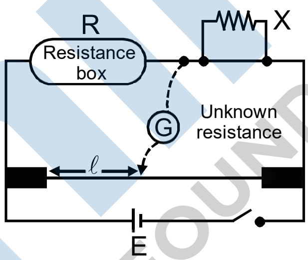
From Maxwell equation

$$\vec{V}\times\vec{E}=-\frac{\partial B}{\partial t}$$

i.e. $\frac{\partial E}{\partial Z}=-\frac{\partial B}{dt}$ and $B_0=\frac{E_0}{C}$

72. In a meter bridge experiment, the circuit diagram and the corresponding observation table are shown in figure.

Sl. No.	R(Ω)	ℓ(cm)
1.	1000	60
2.	100	13
3.	10	1.5
4.	1	1.0



Which of the readings is inconsistent?

(1) 3 (2*) 4 (3) 1 (4) 2

Sol. as $x=\frac{R(100-\ell)}{\ell}$

for (A) $x=\frac{1000\times(100-60)}{40}\approx 667$

for (B) $x=\frac{100\times(100-13)}{13}\approx 669$

for C $x=\frac{10\times(100-1.5)}{98.5}\approx 656$

for D $x=\frac{1\times(100-1)}{1}\approx 95$

So reading in serial no. (4) is completely different hence correct answer (2).

73. The displacement of a damped harmonic oscillator is given by $x(t) = e^{-0.1t}\cos(10pt+\varphi)$. Here t is in seconds. The time taken for its amplitude of vibration to drop to half of its initial value is close to :

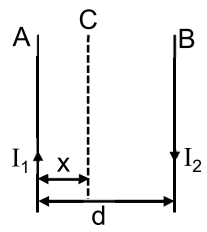
(1*) 7 s (2) 4 s (3) 27 s (4) 13 s

Sol. $A=A_0e^{-0.1t}=\frac{A_0}{2}$

$\ell n2 = 0.1\ t$

$t = 10\ \ell n2 = 6.93 \approx 7$ sec.

74. Two wires A & B are carrying currents I_1 & I_2 as shown in the figure. The separation between them is d. A third wire C carrying a current I is to be kept parallel to them at a distance x from A such that the net force acting on it is zero. The possible values of x are :



(1) $x = \left(\frac{I_2}{I_1+I_2}\right)d$ and $x = \left(\frac{I_2}{I_1-I_2}\right)d$

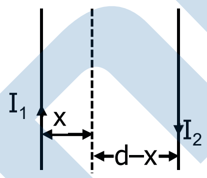
(2*) $x = \pm\frac{I_1 d}{I_1-I_2}$

(3) $x = \left(\frac{I_1}{I_1-I_2}\right)d$ and $x = \left(\frac{I_2}{I_1+I_2}\right)d$

(4) $x = \left(\frac{I_1}{I_1+I_2}\right)d$ and $x = \left(\frac{I_2}{I_1-I_2}\right)d$

Sol. Net force on wire carrying current I per unit length is

$$\frac{\mu_0 I_1 I}{2\pi x} + \frac{\mu_0 I_2 I}{2\pi(d-x)} = 0$$



$$\frac{I_1}{x} = \frac{I_2}{x-d}$$

$$\Rightarrow x = \frac{I_1 d}{I_1 - I_2}$$

75. A uniformly charged ring of radius 3a and total charge q is placed in xy-plane centred at origin. A point charge q is moving towards the ring along the z-axis and has speed υ at z = 4a. The minimum value of υ such that it crosses the origin is :

(1) $\sqrt{\frac{2}{m}}\left(\frac{1}{15}\frac{q^2}{4\pi\varepsilon_0 a}\right)^{1/2}$ (2) $\sqrt{\frac{2}{m}}\left(\frac{1}{5}\frac{q^2}{4\pi\varepsilon_0 a}\right)^{1/2}$ (3*) $\sqrt{\frac{2}{m}}\left(\frac{2}{15}\frac{q^2}{4\pi\varepsilon_0 a}\right)^{1/2}$ (4) $\sqrt{\frac{2}{m}}\left(\frac{4}{15}\frac{q^2}{4\pi\varepsilon_0 a}\right)^{1/2}$

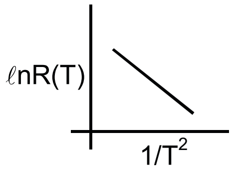
Sol. $U_i + K_i = U_f + K_f$

$$\frac{kq^2}{\sqrt{16a^2+9a^2}} + \frac{1}{2}mv^2 = \frac{kq^2}{3a}$$

$$\frac{1}{2}mv^2 = \frac{kq^2}{a}\left(\frac{1}{3}-\frac{1}{5}\right) = \frac{2kq^2}{15a}$$

$$v = \sqrt{\frac{4kq^2}{15ma}}$$

76. In an experiment, the resistance of a material is plotted as a function of temperature (in some range). As shown in the figure, it is a straight line.



One may conclude that :

(1*) $R(T) = R_0 e^{-T_0^2/T^2}$ (2) $R(T) = \frac{R_0}{T^2}$ (3) $R(T) = R_0 e^{T^2/T_0^2}$ (4) $R(T) = R_0 e^{-T^2/T_0^2}$

Sol. $\frac{\frac{1}{T^2}}{\frac{1}{T_0^2}} + \frac{\text{In}R(T)}{\text{In}R(T_0)} = 1$

$\Rightarrow \text{In } R(T) = \text{In } R(T_0)\left(1 - \frac{T_0^2}{T^2}\right)$

$R(T) = R_0 e^{-\left(\frac{T_0^2}{T^2}\right)}$

77. An npn transistor operates as a common emitter amplifier, with a power gain of 60 dB. The input circuit resistance is 100Ω and the output load resistance is 10 kΩ. The common emitter current gain β is :

(1) 6×10^2 (2) 60 (3) 10^4 (4*) 10^2

Sol. $A_v \times \beta = P_{gain}$

$60 = 10\log\left(\frac{P}{P_0}\right)$

$P = 10^6 = \beta^2 \times \frac{R_{out}}{R_{in}}$

$= \beta \times \frac{10^4}{100}$

$\beta^2 = 10^4$; $\beta = 100$

78. Given below in the the left column are different modes of communication using the kinds of waves given the right column.

A.	Optical Fibre communication	P.	Ultrasound
B.	Radar	Q.	Infrared Light
C.	Sonar	R.	Microwaves
D.	Mobile Phones	S.	Radio Waves

From the options given below, find the most appropriate match between entries in the left and the right column.

(1) A-R, B-P, C-S, D-Q (2) A-S, B-Q, C-R, D-P

(3) A-Q, B-S, C-R, D-P (4*) A-Q, B-S, C-P, D-R

Sol. Factual.

79. A proton, an electron, and a Helium nucleus, have the same energy. They are in circular orbits in a plane due to magnetic field perpendicular to the plane. Let r_p, r_e and r_{He} be their respective radii, then,

(1) $r_e > r_p = r_{He}$ (2) $r_e > r_p > r_{He}$

(3*) $r_e < r_p = r_{He}$ (4) $r_e < r_p < r_{He}$

Sol. $r = \frac{mv}{qB} = \frac{\sqrt{2mK}}{qB}$

$r_{He} = r_p > r_e$

80. Two radioactive materials A and B have decay constants 10λ and λ, respectively. If initially they have the same number of nuclei, then the ratio of the number of nuclei of A to that of B will be 1/e after a time:

(1) $\frac{11}{10\lambda}$ (2) $\frac{1}{10\lambda}$ (3) $\frac{1}{11\lambda}$ (4*) $\frac{1}{9\lambda}$

Sol. $N_1 = N_0e^{-10\lambda t}$; $N_2 = N_0e^{-\lambda t}$

$\frac{1}{e} = \frac{N_1}{N_2} = e^{-9\lambda t}$

$\Rightarrow 9\lambda t = 1$

$\Rightarrow t = \frac{1}{9\lambda}$

81. A stationary source emits sound waves of frequency 500 Hz. Two observers moving along a line passing through the source detect sound to be of frequencies 480 Hz and 530 Hz. Their respective speeds are, in ms^{-1}, (Given speed of sound = 300 m/s)

(1)12,16 (2) 8 ,18 (3) 16, 14 (4*) 12, 18

Sol. $\nu = \frac{v + v_0}{v}\nu_0$

$\Rightarrow v_0 = \left(\frac{\nu}{\mu_0} - 1\right)v$

$v_0 = \left(\frac{530}{500} - 1\right)300 = 18 \text{ m/s}$

$v_0 = \left|\left(\frac{480}{500} - 1\right)300\right| = 12 \text{ m/s}$

82. A moving coil galvanometer allows a full scale current of 10^{-4}A. A series resistance of $2M\Omega$ is required to convert the above galvanometer into a voltmeter of range 0-5 V. Therefore the value of shunt resistance required to convert the above galvanometer into an ammeter of range 0-10 mA is :

(1) 100Ω (2) 200Ω (3) 500Ω (4) 10Ω

Ans. **Bonus**

Sol. $200 + 10^{-4}\,G = 5$
$G = -ve$

So, answer is Bonus.

83. The ratio of surface tensions of mercury and water is given to be 7.5 while the ratio of their densities is 13.6. Their contact angles, with glass, are close to 135° and 0°, respectively. It is observed that mercury gets depressed by an amount h in a capillary tube of radius r_1, while water rises by the same amount h in a capillary tube of radius r_2. The ratio, (r_1/r_2), is then close to :

(1) 3/5 (2*) 2/5 (3) 4/5 (4) 2/3

Sol. $h = \frac{2S_1 \cos\theta}{r_1\rho_1 g}$

$h = \frac{2S_2 \cos\theta_2}{r_2\rho_2 g}$

$\Rightarrow \frac{r_1}{r_2} = \frac{2}{5}$

84. A 25 × 10^{-3} m^3 volume cylinder is filled with 1 mol of O_2 gas at room temperature (300K). The molecular diameter of O_2, and its root mean square speed, are found to be 0.3 nm, and 200 m/s, respectively. What is the average collision rate (per second) for an O_2 molecule ?

(1) ~10^{10} (2*) ~10^{12} (3) ~10^{13} (4) ~10^{11}

Sol. $\nu = \frac{V_{av}}{\lambda}$

$\lambda = \frac{RT}{\sqrt{2}\pi\sigma^2 N_A P}$

$\sigma = 2 \times 0.3 \times 10^{-9}$

$P = \frac{RT}{V}$

$\Rightarrow \quad = \frac{V}{\sqrt{2}\pi\sigma^2 N_A P}$

$V_{av} = \sqrt{\frac{8}{3\pi}} \times V_{rms}$

$\therefore \quad v = \frac{200 \times \sqrt{2}\pi \times \sigma^2 N_A}{25 \times 10^{-3}} \times \sqrt{\frac{8}{3\pi}}$

= 17.68 × 10^8/sec

0.1768 × 10^{10}/sec ~ 10^{10}

This answer does not match with JEE answer key

85. n moles of an ideal gas with constant volume heat capacity C_V undergo an isobaric expansion by certain volume. The ratio of the work done in the process, to the heat supplied is :

(1*) $\frac{nR}{C_V + nR}$ (2) $\frac{nR}{C_V - nR}$ (3) $\frac{4nR}{C_V + nR}$ (4) $\frac{4nR}{C_V - nR}$

Sol. w = nRΔT

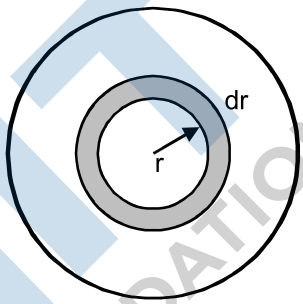
ΔH = (C_v + nR)ΔT

$$\frac{\omega}{\Delta H} = \frac{nR}{C_v + nR}$$

86. A thin disc of mass M and radius R has mass per unit area $\sigma(r) = kr^2$ where r is the distance from its centre. Its moment of inertia about an axis going through its centre of mass and perpendicular to its plane is :

(1) $\frac{MR^2}{6}$ (2*) $\frac{2MR^2}{3}$ (3) $\frac{MR^2}{3}$ (4) $\frac{MR^2}{2}$

Sol. $I_{Disc} = \int_0^R (dm)r^2 \Rightarrow I_{Disc} = \int_0^R (\sigma 2\pi r dr) r^2$



$I_{Disc} = \int_0^R (kr^2 2\pi r dx) r^2$ Mass of disc

$I_{Disc} = 2\pi k \int_0^R r^2 dr$ $M = \int_0^R 2\pi r dr\, kr^2$

$I_{Disc} = 2\pi k \left(\frac{r^6}{6}\right)_0^R$ $M = 2\pi k \int_0^R r^2 dr$

$I_{Disc} = 2\pi k \frac{R^6}{6}$ $M = 2\pi k \left.\frac{r^4}{4}\right|_0^R$

$I_{Disc} = \frac{\pi k R^6}{3} = \left(\frac{\pi k R^4}{2}\right)\frac{R^2 2}{3}$ $M = 2\pi k \left.\frac{r^4}{4}\right|_0^R$

$I_{Disc} = \frac{M2R^2}{3}$; $I_{Disc} = \frac{2}{3}MR^2$

87. A current of 5 A passes through a copper conductor (resistivity = 1.7 × 10^{-8} Ωm) of radius of cross-section 5 mm. Find the mobility of the charges if their drift velocity is 1.1 × 10^{-3} m/s.

(1) 1.8 m^2/Vs (2*) 1.0 m^2/Vs (3) 1.3 m^2/Vs (4) 1.5 m^2/Vs

Sol. $\mu = \frac{V_d}{E}$ $E = \rho J$

$$= \frac{1.1 \times 10^{-13}}{1.7 \times 10^{-8} \times \frac{5}{\pi \times 25 \times 10^{-6}}}$$

$$= \frac{1.1 \times 10^{-13} \times \pi \times 25 \times 10^{-6}}{1.7 \times 10^{-8} \times 5} \approx 1.01\, m^2 / Vs$$

88. A ball is thrown upward with an initial velocity V_0 from the surface of the earth. The motion of the ball is affected by a drag force equal to mv^2 (where m is mass of the ball, v is its instantaneous velocity and γ is a constant) Time taken by the ball to rise to its zenith is :

(1) $\frac{1}{\sqrt{\gamma g}}\ln\left(1+\sqrt{\frac{\gamma}{g}}V_0\right)$

(2) $\frac{1}{\sqrt{\gamma g}}\sin^{-1}\left(\sqrt{\frac{\gamma}{g}}V_0\right)$

(3) $\frac{1}{\sqrt{2\gamma g}}\tan^{-1}\left(\sqrt{\frac{2\gamma}{g}}V_0\right)$

(4*) $\frac{1}{\sqrt{\gamma g}}\tan^{-1}\left(\sqrt{\frac{\gamma}{g}}V_0\right)$

Sol. $-(g+\gamma v^2)=\frac{dv}{dt}$

$$-gdt=\frac{g}{\gamma}\left(\frac{dv}{\frac{g}{\gamma}+v^2}\right)$$

Integrating $0\to t$ and $V_0\to 0$:

$$-gdt=\sqrt{\frac{g}{\gamma}}\tan^{-1}\left(\frac{V_0}{\sqrt{\frac{g}{\gamma}}}\right)$$

$$t=\frac{1}{\sqrt{\gamma g}}\tan^{-1}\left(\sqrt{\frac{\gamma}{g}}V_0\right)$$

89. A cylinder with fixed capacity of 67.2 lit contains helium gas at STP. The amount of heat needed to raise the temperature of the gas by 20°C is : [Given that R = 8.31 J mol^{-1} K^{-1}]

(1) 350 J (2) 374 J (3*) 748 J (4) 700 J

Ans. $\Delta Q = nC_v\Delta T = n\frac{3}{2}R\Delta T$

$$=\left(\frac{67.2}{22.4}\right)\left(\frac{3}{2}\times 8.31\right)(20)$$

≈ 748 J

90. A message signal of frequency 100 MHz and peak voltage 100 V is used to execute amplitude modulation on a carrier wave of frequency 300 GHz and peak voltage 400 V. The modulation index and difference between the two side band frequencies are :

(1) 4 ; 2×10^8 Hz (2) 0.25 ; 1×10^8 Hz (3) 4 ; 1×10^8Hz (4*) 0.25 ; 2×10^8 Hz

Sol. f_m = 100 MHz = 10^8 Hz, $(V_m)_0$ = 100 V

f_c = 300 GHz, $(V_c)_0$ = 400 V

$\therefore$ UBF – LBF = $2f_m$ = 2×10^8 Hz